MOAA 2025: Gunga Bowl

October 11th, 2025

Gunga Bowl Problems

Gunga Bowl Set 1

G1. **[8]** Evaluate

$$\frac{\sqrt{2025}}{5}+\frac{2025^2}{\sqrt{2025^3}}.$$

G2. **[8]** Paige has 5 times as many labubus as Bill. If Bill steals 3 of Paige's labubus, Paige will only have 3 times as many labubus as Bill. How many labubus did Paige have originally?

G3. **[8]** Angela wants to choose 3 Alphastar classes from a catalog of 3 math classes, 4 physics classes. and 2 computer science classes. She wants to choose classes from at least two subject areas, and she must choose a math class. How many ways she can select her classes?

Gunga Bowl Set 2

G4. **[10]** While waiting for his Expii class, Gunga begins drawing a square of side length 1, then a circle of radius 1, then a square of side length 2, then a circle of radius 2, and so on. In general, for each positive integer k, he draws a square of side length k followed by a circle of radius k. He continues this process until the total area of all the shapes he has drawn exceeds 5000. How many shapes does Gunga draw in total?

G5. **[10]** Suppose $\underline{a}\,\underline{b}_7$ is a two-digit base-7 number with $a \neq 0$, such that its value in base 10 is equal to the two-digit number $\underline{b}\,\underline{a}_{10}$ obtained by reversing its digits. Find the sum of all such numbers, expressed in base 10.

G6. **[10]** At the MehtA+ summer camp, there are 2025 students, and numerous clubs. Each student can join any number of clubs, but no two students can be in exactly the same set of clubs. What is the minimum number of clubs at the summer camp?

Gunga Bowl Set 3

G7. **[12]** A quadrilateral is drawn so that all four vertices lie on lattice points, and no other lattice points lie on any of its sides. If the quadrilateral has an area of 18 square units, how many lattice points lie strictly within its boundaries?

G8. **[12]** Sophia and Ellie are standing at points $(1, 4)$ and $(8, 12)$, respectively, in the Cartesian plane. There is a wall along the line $y = \frac{x}{2} + 1$. Sophia throws a ball so that it travels in a straight line, bounces off the wall and then continues in a straight line to Ellie. Find the distance that the ball travels.

G9. **[12]** Satabhisha is standing at the origin of the Cartesian plane. Every minute, she picks a random direction to walk, either up, down, left, or right, and moves one unit in that direction. After five minutes she is on (x, y). If the probability that $|x| + |y| = 5$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime integers, find $m + n$.

Gunga Bowl Set 4

G10. **[14]** The minimum value of the expression

$$\frac{x^3}{y} + \frac{y^3}{z} + \frac{z^3}{x},$$

for positive real numbers x, y, z where $x + y + z = 1$, can be written as a fraction $\frac{p}{q}$, where p and q are relatively prime integers. Find $p + q$.

G11. **[14]** Find the smallest positive integer k such that

$$1^{2025} + 2^{2025} + \ldots + k^{2025}$$

is divisible by 2025.

G12. **[14]** Triangle ABC is inscribed in circle ω. Let M be the midpoint of side AC, and let the median BM be extended to intersect ω again at E. Given that $AB = 5, BC = 3$, and $AC = 6$, the value of ME can be written as $\frac{p}{q}\sqrt{r}$ where p and r are relatively prime integers, and q is not divisible by the square of any prime. Find $p + q + r$.

Gunga Bowl Set 5

G13. **[16]** Carolyn the frog is hopping along the number line. She starts at the number 0, and stops hopping once she reaches the number 100. If her current position is the integer k, she will randomly hop to a new integer position between $k + 1$ and 100, inclusive. Given the probability that Carolyn lands on 75 at some point can be expressed as $\frac{m}{n}$ where m and n are relatively prime integers, find $m + n$.

G14. **[16]** Find the number of quartic polynomials $P(x)$ with integer coefficients satisfying

$$P(1) = 1, P(2) = 4, P(3) = 9, P(4) = 16,$$

and $|P(6)| < 1000$.

G15. **[16]** Let $ABCD$ be a square such that all four of its vertices lie on the graph of

$$2025y^2 = |2024x + 2025|.$$

If AB can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers, find $m + n$.

Gunga Bowl Set 6

G16. **[18]** Consider the sequence of integers a_n which satisfies $a_1 = 1$, $a_2 = 2$, and

$$a_{k+1} = 2a_k - 23a_{k-1}.$$

Find the remainder when a_{2026} is divided by 2024.

G17. **[18]** Each point in a 16×16 lattice is independently colored red with probability p or blue with probability $1 - p$. Let X denote the number of ordered quadruples (P_1, P_2, P_3, P_4) of distinct points such that:

- P_1 and P_2 lie in the same column,
- P_2, P_3, P_4 lie in the same row,
- P_1,P_3, P_4 are red, and P_2 is blue.

Compute the maximum expected value of X over all values of p.

G18. **[18]** A polynomial $p(x)$ with real coefficients satisfies

$$p(p(x)) = 7p(x) - p(-7x) + 2025.$$

Find the sum of all possible values of $|p(1)|$.

Gunga Bowl Set 7

G19. **[20]** Suppose $x, y, z > 0$ are real numbers that satisfy

$$\frac{x+1}{1+y} + \frac{y+1}{1+z} + \frac{z+1}{1+x} = \frac{10}{3}$$
$$\frac{x+1}{1+z} + \frac{y+1}{1+x} + \frac{z+1}{1+y} = \frac{41}{12}$$
$$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = \frac{9}{5}.$$

If $x + y + z$ can be written as $\frac{m}{n}$ where m and n are relatively prime integers, find $m + n$.

G20. **[20]** Let S be a (possibly empty) subset of the integers from 1 to 14, inclusive. Oliver is walking along the number line, starting from the origin. At each moment, he moves 1 or 2 units in the positive direction. However, he does not want to visit any number that is in S. For how many sets S is it possible for Oliver to reach the point 15?

G21. **[20]** How many 6-digit positive integers, containing only the digits 1, 2, 3, and 4, have the property that the sum of the first two digits is less than the sum of the next two digits, which is also less than the sum of the last two digits?

Gunga Bowl Set 8

G22. **[22]** Let n and k be positive integers. Define $G(n, k)$ as the number of functions f from $\{0, 1, 2, \ldots, n\}$ to the integers such that $f(0) = 0$, $f(n) = n(k-2)$, and

$$|f(x) - f(x-1)| \leq k$$

for all $x \in \{1, 2, \ldots, n\}$. Given that $G(n, k) > 2025$, find the smallest possible value of kn.

G23. **[22]** Let $ABCDEF$ be a hexagon inscribed in a circle ω. Lines AD and CE intersect at P, lines CF and AE intersect at Q, and lines EB and AC intersect at R. Given that $AC = 8$, $CE = 6$, and $EA = 10$, and that $ABCDEF$ is the unique hexagon which minimizes

$$\frac{AD}{DP} + \frac{CF}{FQ} + \frac{EB}{BR},$$

find the area of $ABCDEF$.

G24. **[22]** Compute the number of positive integers x and y satisfying

$$2x^3 \equiv y^4 \pmod{2025}$$

where $1 \leq x \leq 2025$ and $1 \leq y \leq 2025$.

Gunga Bowl Set 9

This set consists of three estimation problems, with scoring schemes described.

G25. **[30]** Estimate N, the total number of participants (in person and online) at MOAA this year.

An estimate of e gets a total of $\max\left(0, \left\lfloor 150\left(1 - \frac{|N-e|}{N}\right)\right\rfloor - 120\right)$ points.

G26. **[30]** Estimate L, the total number of letters in all the teams that signed up for MOAA this year, both in person and online.

An estimate of e gets a total of $\max\left(0, \left\lfloor 150\left(1 - \frac{|N-e|}{N}\right)\right\rfloor - 120\right)$ points.

G27. **[30]** Estimate S, the sum of the correct answers to every question across every round of MOAA 2025 (besides Gunga Bowl Set 9).

An estimate of e gets a total of $\max\left(0, 30 - \left\lfloor \frac{|S-e|}{2000} \right\rfloor\right)$ points.